Physical Implications of a Vector-like Extension of the Standard Model[*]

Kazuo Fujikawa

Department of Physics, University of Tokyo

Bunkyo-ku, Tokyo 113, Japan

Physical implications of a vector-like extension of the standard model for heavier quarks and leptons with $SU(2) \times U(1)$ gauge symmetry and only one Higgs doublet are discussed. This scheme incorporates infinitely many fermions, and the chiral structure is realized by a non-vanishing analytic index of mass matrices. The model is perturbatively well controllable, and the physical Higgs particle generally mediates leptonic as well as quark flavor-changing processes at a rate below the present experimental limit. This model illustrates a general feature of models where some of the fermion masses are of non-Higgs origin.

I. INTRODUCTION

An abrupt end of proliferation of quarks and leptons(in particular, light neutrinos) at the 3rd generation is rather mysterious. On the other hand, the weak coupling of the top quark is known to be almost purely left-handed on the basis of the ρ-parameter analysis[1] and also by an analysis of the $b \rightarrow s\gamma$ decay[2]. One of the reasons why the generations with heavier masses are prohibited may be the dynamical stability of Weinberg-Salam theory with chiral (left-handed) weak couplings. Some time ago, we performed an analysis of this problem associated with heavy fermions[3]. We here discuss this problem on the basis of a concrete model[4].

In the standard model, all the masses of gauge bosons and fermions are generated by the Higgs mechanism. In the following, we examine whether heavier quarks and leptons with masses of non-Higgs origin in TeV region, for example, can be accommodated in the standard model when perturbed by the Higgs mechanism which generates masses for conventional quarks and leptons.

We first recall a consequence of the coupling of W boson to the fermion doublet $\psi_k, k = 1, 2$ generically defined by

$$\mathcal{L} = (1/2)g\bar{\psi}_k(T^a)_{kl}\gamma^\mu(a + b\gamma_5)\psi_l W_\mu^a \tag{1}$$

The longitudinal coupling of W_μ^a, which is related to Higgs mechanism , is studied by replacing $W_\mu^a \rightarrow (2/gv)\partial_\mu S^a(x)$ in (1) with $S^a(x)$ the unphysical Higgs scalar. We then obtain

$$\mathcal{L} = a(\frac{m_k - m_l}{v})\overline{\psi}_k(T^a)_{kl}\psi_l S^a + b(\frac{m_k + m_l}{v})\overline{\psi}_k(T^a)_{kl}\gamma_5\psi_l S^a \tag{2}$$

by using the equations of motion.

The typical mass scale of the Higgs world is

$$v = 247 GeV \tag{3}$$

and thus

$$b(m_k + m_l) \gg v, \ or, \ a|m_k - m_l| \gg v \tag{4}$$

induces a strongly interactiong sector into the standard model. We here regard the situation in (4) as unnatural.In other words, the standard model with chiral couplings does not accommodate fermions with masses much larger than the Higgs scale (3). The heavier fermions in the standard scheme, if they should exist without introducing strong couplings, should thus have almost pure vector-like couplings (i.e., $b \simeq 0$ in (4)) and that the mass of the fermion doublet should be almost degenerate (i.e., $a|m_k - m_l| \leq v$ in (4)). If heavier quarks and leptons satisfy the above conditions, they have no sizable couplings to the Higgs sector. In other words, their masses primarily come from dynamics which is different from the Higgs mechanism in the standard model. Also, the breaking mechanism of SU(2) (both of local as well as custodial) in the standard model is concluded to be a typical phenomenon in the energy scale of v.

II. A VECTOR-LIKE EXTENSION OF THE STANDARD MODEL

We consider an $SU(2){\times}U(1)$ gauge theory written in an abbreviated notation[4]

$$\mathcal{L}_L = \overline{\psi}i\gamma^\mu D_\mu\psi - \overline{\psi}_R M\psi_L - \overline{\psi}_L M^\dagger\psi_R \tag{5}$$

with

$$\not{D} = \gamma^\mu(\partial_\mu - igT^aW_\mu^a - i(1/2)g'Y_L B_\mu) \tag{6}$$

and $Y_L = 1/3$ for quarks and $Y_L = -1$ for leptons. The field ψ in (5) is a column vector consisting of an infinite number of $SU(2)$ doublets, and the infinite dimensional *nonhermitian* mass matrix M satisfies the index condition

$$\dim\ker M = 3, \ \dim\ker M^\dagger = 0 \tag{7}$$

In the explicit "diagonalized" expression of M

$$M = \begin{pmatrix} 0 & 0 & 0 & m_1 & 0 & 0 & .. \\ 0 & 0 & 0 & 0 & m_2 & 0 & .. \\ 0 & 0 & 0 & 0 & 0 & m_3 & .. \\ . & . & . & . & . & . & .. \end{pmatrix} \tag{8}$$

the fermion ψ is written as

$$\psi_L = (1-\gamma_5)/2 \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ . \end{pmatrix}, \quad \psi_R = (1+\gamma_5)/2 \begin{pmatrix} \psi_4 \\ \psi_5 \\ \psi_6 \\ . \\ . \end{pmatrix} \tag{9}$$

We thus have 3 massless left-handed $SU(2)$ doublets ψ_1, ψ_2, ψ_3, and an infinite series of vector-like massive $SU(2)$ doublets $\psi_4, \psi_5, ...$ with masses $m_1, m_2, ..$ as is seen in

$$\begin{aligned} \mathcal{L}_L = &\bar{\psi}_1 i\not{D}(\frac{1-\gamma_5}{2})\psi_1 + \bar{\psi}_2 i\not{D}(\frac{1-\gamma_5}{2})\psi_2 \\ &+\bar{\psi}_3 i\not{D}(\frac{1-\gamma_5}{2})\psi_3 \\ &+\bar{\psi}_4(i\not{D} - m_1)\psi_4 + \bar{\psi}_5(i\not{D} - m_2)\psi_5 + ... \end{aligned} \tag{10}$$

An infinite number of right-handed fermions in a doublet notation are also introduced by(again in an abbreviated notation)

$$\mathcal{L}_R = \overline{\phi}i\gamma^\mu(\partial_\mu - i(1/2)g'Y_R B_\mu)\phi - \overline{\phi}_L M'\phi_R - \overline{\phi}_R (M')^\dagger\phi_L \tag{11}$$

where

$$Y_R = \begin{pmatrix} 4/3 & 0 \\ 0 & -2/3 \end{pmatrix} \tag{12}$$

for quarks and

$$Y_R = \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix} \tag{13}$$

for leptons, and the mass matrix M' satisfies the index condition (7) but in general it may have different mass eigenvalues from those in(8). After the diagonalization of M', ϕ is written as

$$\phi_L = (1-\gamma_5)/2 \begin{pmatrix} \phi_4 \\ \phi_5 \\ \phi_6 \\ . \\ . \end{pmatrix}, \quad \phi_R = (1+\gamma_5)/2 \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ . \end{pmatrix} \tag{14}$$

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Here, ϕ_1, ϕ_2, and ϕ_3 are right-handed and massless, and $\phi_4, \phi_5, ...$ have masses $m_1', m_2',..$

$$\begin{aligned}
\mathcal{L}_R = & \bar{\phi}_1 i \not{D} (\frac{1+\gamma_5}{2}) \phi_1 + \bar{\phi}_2 i \not{D} (\frac{1+\gamma_5}{2}) \phi_2 \\
& + \bar{\phi}_3 i \not{D} (\frac{1+\gamma_5}{2}) \phi_3 \\
& + \bar{\phi}_4 (i \not{D} - m_1') \phi_4 + \bar{\phi}_5 (i \not{D} - m_2') \phi_5 + ...
\end{aligned} \tag{15}$$

with

$$\not{D} = \gamma^\mu (\partial_\mu - i(1/2) g' Y_R B_\mu) \tag{16}$$

The present model is vector-like and manifestly anomaly-free before the breakdown of parity (7);after the breakdown of parity,the model still stays anomaly-free provided that both of M and M' satisfy the index condition (7). Unlike conventional vector-like models with a finite number of components, the present scheme avoids the appearance of a strongly interacting right-handed sector despite of the presence of heavy fermions. A truncation of the present scheme to a finite number of heavy fermions (for example, to only one heavy doublet in ψ) is still consistent, although it is no more called vector-like.

The massless fermion sector in the above scheme reproduces the same set of fermions as in the standard model. However, heavier fermions have distinct features. For example, the heavier fermion doublets with the smallest masses are described by

$$\begin{aligned}
\mathcal{L} = & \overline{\psi}_4 i \gamma^\mu (\partial_\mu - ig T^a W_\mu^a - i(1/2) g' Y_L B_\mu) \psi_4 - m_1 \overline{\psi}_4 \psi_4 \\
& + \overline{\phi}_4 i \gamma_\mu (\partial_\mu - i(1/2) g' Y_R B_\mu) \phi_4 - m_1' \overline{\phi}_4 \phi_4
\end{aligned} \tag{17}$$

The spectrum of fermions is thus *doubled* to be vector-like in the sector of heavy fermions and ,at the same time, the masses of ψ and ϕ become non-degenerate, i.e., $m_1 \neq m_1'$ in general. As a result, the fermion number anomaly is generated only by the first 3 generations of light fermions;the violation of baryon number is not enhanced by the presence of heavier fermions. The masses of heavy doublet components in ψ are degenerate in the present zeroth order approximation. The masses of heavy doublets in ϕ are also taken to be degenerate for simplicity.

In the present scheme we distinguish two classes of chiral symmetry breaking;one which is related to the breaking of gauge symmetry (Higgs mechanism), and the other which is related to the mass of heavier fermions but not related to the breaking of gauge symmetry. The transition from one class of chiral symmetry breaking to the other, which is also accompanied by the transition from chiral to vector-like gauge couplings, is assumed to take place at the mass scale of the order of v in (3). In any case if the $SU(2) \times U(1)$ gauge symmetry should be universally valid regardless of the magnitude of the mass of fermions, just like electromagnetism and gravity, the coupling of heavier fermions is required to become vector-like: Heavy gauge bosons can naturally couple to light fermions, but the other way around imposes a stringent constraint on the chirality of fermions. We are here interested in the possible on-set of heavier fermions at the order of a few TeV, although these vector-like components are often assumed to acquire masses of the order of grand unification scale(Georgi's survival hypothesis[7]).

III. HIGGS PERTURBATION AND LIGHT FERMION MASSES

As for the mass generation of the first 3 generations of quarks and leptons and also the custodial $SU(2)$ breaking of heavier fermions, one may introduce a Yukawa interaction for quarks, for example, in an abbreviated notation

$$\begin{aligned}
\mathcal{L}_Y = & \bar{\psi}_L G_u \varphi \phi_R^{(u)} + \bar{\psi}_L G_d \varphi^c \phi_R^{(d)} \\
& + \bar{\psi}_R G_u' \varphi \phi_L^{(u)} + \bar{\psi}_R G_d' \varphi^c \phi_L^{(d)} + h.c.
\end{aligned} \tag{18}$$

where $\varphi(x)$ is the conventional Higgs doublet (and $\varphi(x)^c$ is its conjugate) , and G_u, G_d, G_u' and G_d' are infinite dimensional coupling matrices acting on ψ and ϕ. Corresponding to the presence of only one W-boson, we here assume the existence of only one Higgs doublet. The fields $\bar{\psi}_L$ or $\bar{\psi}_R$ in (18) stands for the doublets in (5), and $\phi_R^{(u)}$ (or $\phi_L^{(u)}$) and $\phi_R^{(d)}$ (or $\phi_L^{(d)}$), respectively, stand for the upper and lower components of the doublets ϕ in (11). If one retains only the first two terms and their conjugates in (18) and if only the massless components of ψ_L and ϕ_R in (9) and (14) are considered, (18) reduces to the Higgs coupling of the standard model.

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We postulate that the coupling matrices G (which generically include G' hereafter) are such that the interaction (18) is perturbatively well controllable,namely, the typical element of coupling matrices G is bounded by the gauge coupling g,

$$|G| \leq g \tag{19}$$

By this way the masses of the first 3 generations of light fermions are generated from (18) below the mass scale in (3). For the heavier fermions, the interaction (18) introduces the breaking of custodial $SU(2)$ and also fermion mixing. After the conventional $SU(2)$ breaking,

$$\langle \varphi \rangle = v/\sqrt{2} \tag{20}$$

one may diagonalize the mass matrix in (18) together with the mass matrices in (5) and (11).This introduces a generalization of the ordinary fermion mixing matrix . If one assumes a generic situation,

$$m_i, m'_j \gg gv, \quad |m_i - m'_j| \gg gv$$
$$|m_i - m_j| \gg gv, \quad |m'_i - m'_j| \gg gv \tag{21}$$

for any combination of (renormalized) heavy fermion masses m_i and m'_j, the masses of heavier fermions are little modified by the Higgs coupling. As a fiducial value of the on-set of heavy fermion mass, we here choose

$$m_i \sim a\ few\ TeV \tag{22}$$

In practical calculations, it is convenient to diagonalize the light fermion masses in addition to (8) and its analogue of M' but leave the mixing of light and heavy fermions non-diagonalized, instead of diagonalizing all the masses. In this case the effects of heavy fermions on the processes of light fermions are estimated in the power expansion of G. The mass term after diagonalizing the light quarks in the up-quark sector, for example, is given by

$$(\bar{\psi}_L, \bar{\Psi}_L, \bar{\Phi}_L) \mathcal{M} \begin{pmatrix} \phi_R \\ \Psi_R \\ \Phi_R \end{pmatrix} + h.c. \tag{23}$$

where the mass matrix $\mathcal{M}$ has a structure

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 & & 0 & & \tilde{G}v/\sqrt{2} \\ 0 & 0 & m_t \\ & & & m_1 & 0 & 0 \\ & \tilde{G}v/\sqrt{2} & & 0 & m_2 & 0 & & Gv/\sqrt{2} \\ & & & 0 & 0 & .. \\ & & & & & & m'_1 & 0 & 0 \\ & 0 & & & Gv/\sqrt{2} & & 0 & m'_2 & 0 \\ & & & & & & 0 & 0 & .. \end{pmatrix}$$

To avoid introducing further notational conventions, we here use the fields ψ_L and ϕ_R in(23) for the first 3 light (i.e.,massless in the zeroth order approximation) fermion components of ψ and ϕ in (9) and (14),respectively; Ψ and Φ stand for the remaining heavy quark components of ψ and ϕ in (9) and (14). The coupling matrix $\tilde{G}$ is different from the original G by the unitary transformation of light quarks performed in the process of diagonalizing light quark masses. But the order of magnitude of $\tilde{G}$ is still the same as that of G. We note that the Higgs coupling (18) mixes the left- and right-handed fermions and thus leads to a breaking of the index condition (7), which ensured the presence of 3 massless fermions without the Higgs coupling.

The physical Higgs $H(x)$ coupling in the unitary gauge is given by the replacement $v \to v + H(x)$ in the above mass matrix(23).

If one sets $G = 0$ and $\tilde{G} = 0$ in the mass matrix(23), the light and heavy quark sectors become completely disconnected, not only in the Higgs coupling but also in the gauge coupling except for the renormalization effects due to heavy quark loop diagrams. This means that the <u>direct</u> effects of heavy fermions on the processes involving light quarks and leptons only can be calculated as a power series in $\tilde{G}$ and G, provided that these effects of heavy fermions are small. It is shown [4] that these effects are in fact of controllable magnitude if the condition $|G| \leq g$ in (19) is satisfied and the mass spectrum of heavy fermions starts at a few TeV.

Some of the physical implications of the present scheme have benn analyzed [4][5]. Heavier fermions are expected to decay mainly into the Higgs particle and light fermions with natural decay width

$$\Gamma_i \sim |G|^2 m_i. \tag{24}$$

if we take the Higgs mass at the "natural" value $m_H \sim v$ with v in (3).

The mass spectrum of light fermions is influenced by heavier fermions through the mixing in (18), but the most natural choice $|G| \sim g$ already gives a sensible result [4],although a certain fine tuning is required to account for the actual masses of the electron and up and down quarks.

A characteristic feature of the present extension of the standard model is that the leptonic as well as quark flavor is generally violated; this breaking is caused by the mixing of light and heavy fermions in (18). The diagonalization of mass matrix does not diagonalize the Higgs coupling in general unlike the standard model, and the physical Higgs particle at the tree level also mediates flavor changing processes although its contribution is not necessarily a dominant one; this is a rather general feature of a model where some of the fermion masses are of non-Higgs origin. In the limit of large heavy fermion masses $m_i \to \infty$ in the present model, this flavor changing coupling vanishes. The Higgs and heavy fermion contributions to GIM suppressed processes in the standard model are shown to be small below the present experimental limit[4].

The leptonic flavor changing processes such as $K_L^0 \to e\bar{\mu}$ are also induced by the mixing of heavy fermions , as was noted above. The decay rate of $K_L^0 \to e\bar{\mu}$ is then estimated at the order

$$\Gamma(K_L^0 \to e\bar{\mu}) \leq 10^{-8} \times \Gamma(K_L^0 \to \mu\bar{\mu}) \tag{25}$$

where $\Gamma(K_L^0 \to \mu\bar{\mu})$ is given by the standard model.

It is confirmed that CP violation does not appear in the zeroth order approximation without the Higgs coupling (see eq.(10)) and it arises solely in the Higgs sector(18);the pattern of CP violation becomes more involved than in the standard model and CP phase is no more limited to W couplings.

As for the neutrinos, the first three neutrinos will remain massless if one assumes the absence of right-handed components(i.e., if ϕ in (11) is a singlet). We however expect the appearance of vector-like heavy neutrinos above TeV region.

IV. DISCUSSION

A vector-like extension of the standard model examined in this note is natural, in the sense that the validity of perturbation theory (19) combined with a sensible choice of heavy fermion mass scale (23) lead to consistent results as a first order approximation.

The present model as it stands is, however, a phenomenological one:The appearance of many fermions with vector-like couplings might be natural from some kind of composite picture of fermions, or if the fermions are elementary their masses might arise from a topological origin as is suggested by (7) or from some kind of space-time compactification. But the fundamental issue of the breaking mechanism of chiral and parity symmetries by the analytic index in (7) needs to be explained. The breaking of asymptotic freedom of QCD by heavy fermions becomes appreciable only at the mass scale of these heavy fermions due to the decoupling phenomenon.

[1] M. Veltman, Nucl. Phys. **B123**,89(1977).
   R.D.Peccei, S.Peris and X.Zhang, Nucl. Phys. **B349**,305(1991).
[2] K. Fujikawa and A. Yamada, Phys. Rev. **D49**,5890(1994), and references therein.
[3] K. Fujikawa,Prog. Theor. Phys. **61**,1186(1979).
[4] K. Fujikawa, Prog. Theor. Phys. **92**,1149(1994).
   For somewhat related models of fermions from different view points,see
   J. Pati,Phys. Rev. **D51**, 2451(1995).
   K. Inoue, Prog. Theor. Phys. **93**, 403(1995).
   H. Zheng, Phys. Rev. **D52**,6500(1995); hep-ph/9602340.
[5] We here assume that the mass spectrum of heavier fermions is rather sparsely distributed. We thus estimate the effects of the lightest heavier fermions on physical processes involving ordinary fermions in the standard model.

[6] G. 't Hooft, Phys. Rev. Lett. **37**,8(1976).

[7] H. Georgi, Nucl.Phys.**B156**,126(1979).